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Symmetric and asymmetric quantum channels in quantum communication systems

Masashi Ban

Advanced Research Laboratory, Hitachi, Ltd, 2520 Akanuma, Hatoyama, Saitama 350-0395, Japan
and
CREST, Japan Science and Technology Agency, 1-1-9 Yaesu, Chuo-ku, Tokyo 103-0028, Japan

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Abstract

Symmetric and asymmetric quantum channels which act on bipartite bosonic states are considered. The linear dissipative channel and the quantum teleportation channel are applied. The influences of the symmetric and asymmetric quantum channels on bipartite Gaussian states are investigated by means of the inseparability condition. Furthermore, quantum teleportation and quantum dense coding of continuous variables performed by means of two-mode squeezed-vacuum states under the influence of the noisy quantum channels are discussed.

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1. Introduction

Quantum information processing provides the possibilities of novel information technology such as quantum cryptography, quantum communication and quantum computation as well as new insights on the principles of quantum mechanics [1, 2]. Entanglement between quantum systems is one of the most important resources in quantum information processing. To perform quantum communication such as quantum teleportation [3–6] and quantum dense coding [7–11], a sender and a receiver must share quantum entanglement. When quantum entanglement is shared by distant users, the influence of noisy quantum channels on the quantum entanglement is inevitable. Noisy quantum channels cause decoherence and thus they are obstacles to performing quantum communication with high performance. Since the two modes of a bipartite quantum state are sent through noisy quantum channels to share the entanglement, it is reasonable to consider that the quantum channel acting on one mode of the bipartite state is different, in general, from that acting on the other mode. Then it is important to investigate quantum communication under the influence of symmetric and asymmetric quantum channels. In particular, it is interesting to ask whether asymmetry of quantum channels is good or not for sharing quantum entanglement and performing quantum communication.

This paper investigates the influence of symmetric and asymmetric Gaussian quantum channels on the entanglement of bipartite quantum states. In section 2, we briefly review the two quantum channels considered in this paper. One is the linear dissipative channel and the other is the quantum teleportation channel. In section 3, we discuss the inseparability of bipartite Gaussian states transmitted through these quantum channels. In particular, we focus our attention on the asymmetry of the quantum channels. In section 4, we investigate quantum teleportation and quantum dense coding of continuous variables under the influence of the asymmetric Gaussian channels. Concluding remarks are given in section 5.

2. Symmetric and asymmetric quantum channels

This section introduces the linear dissipative channel and the quantum teleportation channel. The Wigner function and the characteristic function of the output states of the quantum channels are obtained. The results are used in the rest of this paper.

2.1. Linear dissipative channel caused by noisy environment

The time evolution of the quantum state $\hat{\rho}(t)$ of a bosonic system under the influence of a noisy environment is determined by the quantum master equation [12]

$$\frac{\partial}{\partial t}\hat{\rho}(t) = \kappa_k\big(\bar{n}_k^{(\mathrm{th})}+1\big)\big[2\hat{a}_k\hat{\rho}(t)\hat{a}_k^\dagger - \hat{a}_k^\dagger\hat{a}_k\hat{\rho}(t) - \hat{\rho}(t)\hat{a}_k^\dagger\hat{a}_k\big] + \kappa_k\bar{n}_k^{(\mathrm{th})}\big[2\hat{a}_k^\dagger\hat{\rho}(t)\hat{a}_k - \hat{a}_k\hat{a}_k^\dagger\hat{\rho}(t) - \hat{\rho}(t)\hat{a}_k\hat{a}_k^\dagger\big] \tag{1}$$

where $\hat{a}_k$ and $\hat{a}_k^\dagger$ are bosonic annihilation and creation operators of the kth mode ($k = 1, 2$). The non-negative parameters κ_k and $\bar{n}_k^{(\mathrm{th})}$ appearing in equation (1) represent the relaxation constant and the average photon number of the thermal noise. In equation (1), we have ignored the term which represents the free time evolution since it is not important for our purpose. Introducing the Wigner function $W(z;t)$ of the quantum state $\hat{\rho}(t)$ by [13, 14]

$$W(z;t) = 2\,\mathrm{e}^{2|z|^2}\int\frac{\mathrm{d}^2\alpha}{\pi}\langle-\alpha|\hat{\rho}(t)|\alpha\rangle\,\mathrm{e}^{-2\alpha z^*+2\alpha^* z} \tag{2}$$

where $|\pm\alpha\rangle$ is the Glauber coherent state and $\mathrm{d}^2\alpha$ stands for $\mathrm{d}(\mathrm{Re}\,\alpha)\,\mathrm{d}(\mathrm{Im}\,\alpha)$, one can derive the Fokker–Planck equation for the Wigner function $W(z;t)$ [12] from equation (1) and can obtain the general solution

$$W(z;t) = \int\frac{\mathrm{d}^2z'}{\pi}G_k(z|z';t)W(z';0) \tag{3}$$

with

$$G_k(z|z';t) = \frac{1}{\big(\bar{n}_k^{(\mathrm{th})}+\frac{1}{2}\big)(1-\mathrm{e}^{-2\kappa_k t})}\exp\left[-\frac{|z-z'\,\mathrm{e}^{-\kappa_k t}|^2}{\big(\bar{n}_k^{(\mathrm{th})}+\frac{1}{2}\big)(1-\mathrm{e}^{-2\kappa_k t})}\right]. \tag{4}$$

In the rest of this paper, we denote the initial state $\hat{\rho}(0)$ and the final state $\hat{\rho}(t)$ at time t as $\hat{\rho}_{\mathrm{in}}$ and $\hat{\rho}_{\mathrm{out}}$, where t stands for the transmission time during which the system passes through the linear dissipative channel.

For the Wigner function $W(z_1, z_2; t)$ of a two-mode bosonic state, one can obtain the input–output relation from equations (3) and (4)

$$W_{\rm out}(z_1, z_2) = \int \frac{{\rm d}^2 z_1'}{\pi} \int \frac{{\rm d}^2 z_2'}{\pi} G_1(z_1|z_1'; t) G_2(z_2|z_2'; t) W_{\rm in}(z_1', z_2'). \tag{5}$$

This relation determines the quantum channel for any two-mode bosonic state. If $\kappa_1 = \kappa_2$ and $\bar{n}_1^{(\rm th)} = \bar{n}_2^{(\rm th)}$, the quantum channel is symmetric and otherwise it is asymmetric. The statistical properties of a quantum state $\hat{\rho}$ can be derived from the characteristic function defined by $C(\alpha) = {\rm Tr}\left[{\rm e}^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}\hat{\rho}\right]$. Since the characteristic function is the Fourier transformation of the Wigner function, the characteristic function $C_{\rm out}(\alpha_1, \alpha_2)$ of the two-mode quantum state at the channel output is obtained from equations (4) and (5)

$$\begin{aligned} C_{\rm out}(\alpha_1, \alpha_2) &= \int \frac{{\rm d}^2 z_1}{\pi} \int \frac{{\rm d}^2 z_1}{\pi} W_{\rm out}(z_1, z_2; t)\, {\rm e}^{\alpha_1 z_1^* - \alpha_1^* z_1}\, {\rm e}^{\alpha_2 z_2^* - \alpha_2^* z_2} \\ &= G_1(\alpha_1; t) G_2(\alpha_2; t) C_{\rm in}(\alpha_1\, {\rm e}^{-\kappa_1 t}, \alpha_2\, {\rm e}^{-\kappa_2 t}) \end{aligned} \tag{6}$$

with

$$G_k(\alpha; t) = \exp[-N_k(t)|\alpha|^2] \tag{7}$$

$$N_k(t) = \left(\bar{n}_k^{(\rm th)} + \tfrac{1}{2}\right)(1 - {\rm e}^{-2\kappa_k t}) \tag{8}$$

where $C_{\rm in}(\alpha_1, \alpha_2)$ is the characteristic function of the initial state $\hat{\rho}_{\rm in} = \hat{\rho}(0)$. When the transmission times t_1 and t_2 of the two modes are different, it is easy to see that equation (6) is generalized to

$$C_{\rm out}(\alpha_1, \alpha_2) = G_1(\alpha_1; t_1) G_2(\alpha_2; t_2) C_{\rm in}(\alpha_1\, {\rm e}^{-\kappa_1 t_1}, \alpha_2\, {\rm e}^{-\kappa_2 t_2}). \tag{9}$$

This result will be used for investigating the inseparability of bipartite Gaussian states sent through the symmetric and asymmetric linear dissipative channels.

2.2. *Quantum channel equivalent to quantum teleportation*

Quantum teleportation is a novel method for transmitting an unknown quantum state by means of classical communication with the assistance of quantum entanglement. Hence it is equivalent to the quantum channel. When continuous variable quantum teleportation with the standard protocol [5, 15–18] is performed by sharing a bipartite quantum state $\hat{\Psi}_k$, any input state $\hat{\rho}_{\rm in}$ to be teleported is transformed into the output state $\hat{\rho}_{\rm out}$ which is related to the input state $\hat{\rho}_{\rm in}$ by [15–18]

$$\hat{\rho}_{\rm out} = \int \frac{{\rm d}^2\alpha}{\pi} P_k(\alpha) \hat{D}(\alpha) \hat{\rho}_{\rm in} \hat{D}^\dagger(\alpha) \tag{10}$$

with

$$P_k(\alpha) = \int \frac{{\rm d}^2\beta}{\pi} W_{\Psi_k}(\beta^* - \alpha^*, \beta) \tag{11}$$

where $W_{\Psi_k}(z_1, z_2)$ is the Wigner function of the bipartite quantum state $\hat{\Psi}_k$. The standard protocol implies that the sender performs a simultaneous measurement of position and momentum and the receiver applies the displacement operator, the amplitude of which is determined by the measurement outcome obtained from the sender via a classical communication channel.

We assume that the bipartite quantum state $\hat{\Psi}_k$ is the Gaussian state which is called the mixed EPR state [19, 20], the characteristic function of which is the Gaussian distribution

$$C_{\Psi_k}(z_1, z_2) = \exp\left(-\tfrac{1}{2}\mathsf{z}^\dagger \mathsf{V}_k \mathsf{z}\right) \tag{12}$$

where $\mathsf{z} = (z_1^*, z_1, z_2^*, z_2)^\dagger$ and the Hermitian matrix V_k is given by

$$\mathsf{V}_k = \begin{pmatrix} \bar{n}_k + \frac{1}{2} & 0 & 0 & \bar{m}_k \\ 0 & \bar{n}_k + \frac{1}{2} & \bar{m}_k^* & 0 \\ 0 & \bar{m}_k & \bar{n}_k + \frac{1}{2} & 0 \\ \bar{m}_k^* & 0 & 0 & \bar{n}_k + \frac{1}{2} \end{pmatrix}. \tag{13}$$

The parameters $\hat{n}_k$ and $\bar{m}_k$ must satisfy the inequality $\bar{n}_k(\bar{n}_k + 1) \geqslant |\bar{m}|^2$ due to the uncertainty relation. In this case, the function $P(\alpha)$ given by equation (11) becomes

$$P(\alpha) = \frac{1}{\Delta_{\bar{n}_k\bar{m}_k}} \exp\left(-\frac{|\alpha|^2}{\Delta_{\bar{n}_k\bar{m}_k}}\right) \tag{14}$$

with positive parameter $\Delta_{\bar{n}_k\bar{m}_k} = 2\bar{n}_k + \bar{m}_k + \bar{m}_k^* + 1$. Hence it is found that continuous variable quantum teleportation reduces to the quantum thermalizing channel

$$\hat{\rho}_{\text{out}} = \frac{1}{\Delta_{\bar{n}_k\bar{m}_k}} \int \frac{\mathrm{d}^2\alpha}{\pi} \, \mathrm{e}^{-|\alpha|^2/\Delta_{\bar{n}_k\bar{m}_k}} \hat{D}(\alpha)\hat{\rho}_{\text{in}}\hat{D}^\dagger(\alpha). \tag{15}$$

This channel is also derived when the external environment randomly modulates the complex amplitude of the quantum state and the basic properties have been investigated in detail [21]. Using the Wigner functions $W_{\text{in}}(z)$ and $W_{\text{out}}(z)$ of the input and output states, one can obtain the input–output relation of the Wigner function

$$W_{\text{out}}(z) = \int \frac{\mathrm{d}^2 z'}{\pi} \mathcal{G}_k(z|z') W_{\text{in}}(z') \tag{16}$$

with

$$\mathcal{G}_k(z|z') = \frac{1}{\Delta_{\bar{n}_k\bar{m}_k}} \exp\left(-\frac{|z - z'|^2}{\Delta_{\bar{n}_k\bar{m}_k}}\right). \tag{17}$$

We suppose that quantum teleportation of a two-mode state is performed, where each mode is teleported by means of the bipartite Gaussian state $\hat{\Psi}_k$ characterized by equations (12) and (13). In this case, the Wigner function $W_{\text{out}}(z_1, z_2)$ of the output state is related to the Wigner function $W_{\text{in}}(z_1, z_2)$ of the input state by

$$W_{\text{out}}(z_1, z_2) = \int \frac{\mathrm{d}^2 z_1'}{\pi} \int \frac{\mathrm{d}^2 z_2'}{\pi} \mathcal{G}_1(z_1|z_1')\mathcal{G}_2(z_2|z_2') W_{\text{in}}(z_1', z_2'). \tag{18}$$

Here it is obvious that if $\hat{\Psi}_1 = \hat{\Psi}_2$, the quantum channel is symmetric and otherwise it is asymmetric. Using the characteristic functions $C_{\text{in}}(\alpha_1, \alpha_2)$ and $C_{\text{out}}(\alpha_1, \alpha_2)$ of the input and output states, one can obtain the relation from equation (18)

$$C_{\text{out}}(\alpha_1, \alpha_2) = \mathcal{G}_1(\alpha_1)\mathcal{G}_2(\alpha_2) C_{\text{in}}(\alpha_1, \alpha_2) \tag{19}$$

with $\mathcal{G}_k(\alpha) = \exp\left(-\Delta_{\bar{n}_k\bar{m}_k}|\alpha|^2\right)$. When the kth mode of the input state is teleported τ_k times by means of the identical bipartite Gaussian states $\hat{\Psi}_k$ [23], the input–output relation for the Wigner functions is generalized to

$$W_{\text{out}}(z_1, z_2) = \int \frac{\mathrm{d}^2 z_1'}{\pi} \int \frac{\mathrm{d}^2 z_2'}{\pi} \mathcal{G}_1(z_1|z_1'; \mu_1)\mathcal{G}_2(z_2|z_2'; \mu_2) W_{\text{in}}(z_1', z_2') \tag{20}$$

with

$$\mathcal{G}_k(z|z'; \mu_k) = \frac{1}{\tau_k \Delta_{\bar{n}_k\bar{m}_k}} \exp\left(-\frac{|z - z'|^2}{\tau_k \Delta_{\bar{n}_k\bar{m}_k}}\right). \tag{21}$$

The characteristic function of the output state is given by

$$C_{\text{out}}(\alpha_1, \alpha_2) = \mathcal{G}_1(\alpha_1; \tau_1)\mathcal{G}_2(\alpha_2; \tau_2) C_{\text{in}}(\alpha_1, \alpha_2) \tag{22}$$

with

$$\mathcal{G}_k(\alpha;\tau) = \exp[-\mathcal{N}_k(\tau)|\alpha|^2] \tag{23}$$

$$\mathcal{N}_k(\tau) = \tau\Delta_{\bar{n}_k\bar{m}_k} = \tau(2\bar{n}_k + \bar{m}_k + \bar{m}_k^* + 1). \tag{24}$$

It is obvious from equations (6) and (22) that both the linear dissipative channel and the quantum teleportation channel transform any Gaussian state into another Gaussian state. Thus these quantum channels belong to the set of Gaussian channels. The properties of Gaussian channels have been investigated in detail [22].

3. Inseparability of output Gaussian states

This section investigates the inseparability of the output states of the linear dissipative channels and the quantum teleportation channels when the input states are bipartite Gaussian states $\hat{\rho}_G$. First we briefly summarize the necessary and sufficient condition for the separability of bipartite Gaussian states. The characteristic function of an arbitrary bipartite Gaussian state can be written as [20]

$$C_{\rm in}(z_1, z_2) = \mathrm{Tr}\left[\hat{\rho}_G\left(\mathrm{e}^{z_1\hat{a}_1^\dagger - z_1^*\hat{a}_1} \otimes \mathrm{e}^{z_2\hat{a}_2^\dagger - z_2^*\hat{a}_2}\right)\right] = \exp\left(-\tfrac{1}{2}\mathsf{z}^\dagger \mathsf{V}_{\rm in}\mathsf{z}\right) \tag{25}$$

where $\mathsf{z} = (z_1^*, z_1, z_2^*, z_2)^\dagger$ and the Hermitian 4×4 matrix $\mathsf{V}_{\rm in}$ is given by

$$\mathsf{V}_{\rm in} = \begin{pmatrix} \mathsf{V}_{11} & \mathsf{V}_{12} \\ \mathsf{V}_{12}^\dagger & \mathsf{V}_{22} \end{pmatrix} \tag{26}$$

and the 2×2 matrices V_{kk} $(k = 1, 2)$ and V_{12} in $\mathsf{V}_{\rm in}$ are given by

$$\mathsf{V}_{kk} = \begin{pmatrix} \bar{n}_k^{(\rm in)} + \frac{1}{2} & \bar{m}_k^{(\rm in)} \\ \bar{m}_k^{(\rm in)*} & \bar{n}_k^{(\rm in)} + \frac{1}{2} \end{pmatrix} \qquad \mathsf{V}_{12} = \begin{pmatrix} \bar{m}_s^{(\rm in)} & \bar{m}_c^{(\rm in)} \\ \bar{m}_c^{(\rm in)*} & \bar{m}_s^{(\rm in)*} \end{pmatrix}. \tag{27}$$

Here the parameters are $\bar{n}_k^{(\rm in)} = \langle \hat{a}_k^\dagger \hat{a}_k\rangle$, $\bar{m}_k^{(\rm in)} = -\langle \hat{a}_k^2\rangle$, $\bar{m}_s^{(\rm in)} = \langle \hat{a}_1\hat{a}_2^\dagger\rangle$ and $\bar{m}_c^{(\rm in)} = -\langle \hat{a}_1\hat{a}_2\rangle$. In equation (25), we have ignored the linear term with respect to z in the exponential since it is irrelevant to the inseparability of the bipartite Gaussian state. When the quantum channels are the linear dissipative ones, the characteristic function $C_{\rm out}(z_1, z_2)$ of the output state is obtained from equations (9) and (25)

$$C_{\rm out}(z_1, z_2) = \exp\left(-\tfrac{1}{2}\mathsf{z}^\dagger \mathsf{V}_{\rm out}\mathsf{z}\right) \tag{28}$$

with

$$\mathsf{V}_{\rm out} = \begin{pmatrix} N_1(t_1)\mathsf{I} + \mathsf{V}_{11}\,\mathrm{e}^{-2\kappa_1 t_1} & \mathsf{V}_{12}\,\mathrm{e}^{-\kappa_1 t_1 - \kappa_2 t_2} \\ \mathsf{V}_{12}^\dagger\,\mathrm{e}^{-\kappa_1 t_1 - \kappa_2 t_2} & N_2(t_2)\mathsf{I} + \mathsf{V}_{11}\,\mathrm{e}^{-2\kappa_2 t_2} \end{pmatrix} \tag{29}$$

where I stands for a 2×2 identity matrix and $N_k(t)$ is given by equation (8). On the other hand, for the quantum teleportation channels described by equation (22), the matrix $\mathsf{V}_{\rm out}$ becomes

$$\mathsf{V}_{\rm out} = \begin{pmatrix} \mathcal{N}_1(\tau_1)\mathsf{I} + \mathsf{V}_{11} & \mathsf{V}_{12} \\ \mathsf{V}_{12}^\dagger & \mathcal{N}_2(\tau_2)\mathsf{I} + \mathsf{V}_{11} \end{pmatrix} \tag{30}$$

where $\mathcal{N}_k(\tau_k)$ is given by equation (24). Furthermore, the necessary and sufficient condition for the output Gaussian state to be inseparable is given by [19, 20, 24, 25]

$$\mathsf{T}_1\mathsf{V}_{\rm out}\mathsf{T}_1 + \tfrac{1}{2}\mathsf{E} < 0 \tag{31}$$

where the matrices T_1 are defined by

$$\mathsf{T}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathsf{E} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{32}$$

The matrix T_1 represents the partial transposition with respect to the mode 1.

We suppose that the input bipartite Gaussian state is the mixed EPR state which includes a noisy two-mode squeezed-vacuum state as a special case, where the matrices V_{kk} and V_{12} are given respectively by

$$\mathsf{V}_{kk} = \begin{pmatrix} \bar{n}_k^{(\text{in})} + \frac{1}{2} & 0 \\ 0 & \bar{n}_k^{(\text{in})} + \frac{1}{2} \end{pmatrix} \qquad \mathsf{V}_{12} = \begin{pmatrix} 0 & \bar{m}^{(\text{in})} \\ \bar{m}^{(\text{in})*} & 0 \end{pmatrix} \tag{33}$$

where the uncertainty relation implies that the inequality $\bar{n}_k^{(\text{in})}\left(\bar{n}_k^{(\text{in})} + 1\right) \geqslant |\bar{m}^{(\text{in})}|^2$ $(k = 1, 2)$ holds. In this case, it is found that the input bipartite Gaussian state is inseparable if and only if the parameters $\bar{n}_k^{(\text{in})}$ $(k = 1, 2)$ and $\bar{m}^{(\text{in})}$ satisfy the inequality $\bar{n}_1^{(\text{in})}\bar{n}_2^{(\text{in})} < |\bar{m}^{(\text{in})}|^2$. Moreover the necessary and sufficient condition that the output bipartite Gaussian state of the linear dissipative channels become inseparable is obtained from equations (29) and (31)

$$\left[\left(\bar{n}_1^{(\text{th})} - \bar{n}_1^{(\text{in})}\right)\left(1 - \mathrm{e}^{-2\kappa_1 t_1}\right) + \bar{n}_1^{(\text{in})}\right]\left[\left(\bar{n}_2^{(\text{th})} - \bar{n}_2^{(\text{in})}\right)\left(1 - \mathrm{e}^{-2\kappa_2 t_2}\right) + \bar{n}_2^{(\text{in})}\right] < |\bar{m}^{(\text{in})}|^2 \,\mathrm{e}^{-2(\kappa_1 t_1 + \kappa_2 t_2)}. \tag{34}$$

On the other hand, for the quantum teleportation channels, the necessary and sufficient condition of the inseparability of the output Gaussian state is obtained from equations (30) and (31)

$$\left[\Delta_{\bar{n}_1\bar{m}_1}\tau_1 + \bar{n}^{(\text{in})}\right]\left[\Delta_{\bar{n}_2\bar{m}_2}\tau_2 + \bar{n}^{(\text{in})}\right] < |\bar{m}^{(\text{in})}|^2. \tag{35}$$

In the rest of this section, we will investigate the properties of inequalities (34) and (35) for the symmetric and asymmetric Gaussian channels and will show how the quantum channels affect the entanglement of bipartite Gaussian states.

3.1. Symmetric and asymmetric linear dissipative channels

Using inequality (34), we investigate the inseparability of the bipartite Gaussian state transmitted through the symmetric and asymmetric linear dissipative channels. We find the most useful setup that makes it possible for distant users to share the inseparable Gaussian state. We suppose that the noisy environments are equal for the two modes ($\bar{n}_1^{(\text{th})} = \bar{n}_2^{(\text{th})} \equiv \bar{n}^{(\text{th})}$ and $\kappa_1 = \kappa_2 \equiv \kappa$) while the transmission times t_1 and t_2 are different. Then inequality (34) can be rewritten as

$$(\bar{n}^{(\text{th})}\,\mathrm{e}^{2\kappa t_1} + \Delta\bar{n}_1)(\bar{n}^{(\text{th})}\,\mathrm{e}^{2\kappa t_2} + \Delta\bar{n}_2) < |\bar{m}^{(\text{in})}|^2 \tag{36}$$

where the effects of the linear dissipative channels are included on the left-hand side which is denoted as

$$F(t_1, t_2) = (\bar{n}^{(\text{th})}\,\mathrm{e}^{2\kappa t_1} + \Delta\bar{n}_1)(\bar{n}^{(\text{th})}\,\mathrm{e}^{2\kappa t_2} + \Delta\bar{n}_2) \tag{37}$$

with $\Delta\bar{n}_k = \bar{n}_k^{(\text{in})} - \bar{n}^{(\text{th})}$ $(k = 1, 2)$. To investigate the influences of the quantum channels on the inseparability of the output Gaussian state, we obtain the maximum and minimum values F_{max} and F_{min} of the function $F(t_1, t_2)$ under the constraint that the total transmission time $t = t_1 + t_2$ is fixed. This is equivalent to finding the best and worst places of the generator of the initial Gaussian state between the distant users who would like to share it. Straightforward calculation provides the following result:

(i) if $\bar{n}_1^{(\mathrm{in})} < \bar{n}^{(\mathrm{th})} < \bar{n}_2^{(\mathrm{in})}$,

$$F_{\min} = F_1 \quad \text{at} \quad (t_1, t_2) = (0, t) \tag{38}$$

$$F_{\max} = F_2 \quad \text{at} \quad (t_1, t_2) = (t, 0). \tag{39}$$

(ii) if $\bar{n}_1^{(\mathrm{in})} > \bar{n}^{(\mathrm{th})} > \bar{n}_2^{(\mathrm{in})}$,

$$F_{\min} = F_2 \quad \text{at} \quad (t_1, t_2) = (t, 0) \tag{40}$$

$$F_{\max} = F_1 \quad \text{at} \quad (t_1, t_2) = (0, t). \tag{41}$$

(iii) if $\bar{n}_1^{(\mathrm{in})} \geqslant \bar{n}^{(\mathrm{th})}, \bar{n}_2^{(\mathrm{in})} \geqslant \bar{n}^{(\mathrm{th})}$ and $\Delta\bar{n}_1/\Delta\bar{n}_2 > \mathrm{e}^{2\kappa t}$,

$$F_{\min} = F_2 \quad \text{at} \quad (t_1, t_2) = (t, 0) \tag{42}$$

$$F_{\max} = F_1 \quad \text{at} \quad (t_1, t_2) = (0, t). \tag{43}$$

(iv) if $\bar{n}_1^{(\mathrm{in})} < \bar{n}^{(\mathrm{th})}, \bar{n}_2^{(\mathrm{in})} < \bar{n}^{(\mathrm{th})}$ and $\Delta\bar{n}_1/\Delta\bar{n}_2 > \mathrm{e}^{2\kappa t}$,

$$F_{\min} = F_1 \quad \text{at} \quad (t_1, t_2) = (0, t) \tag{44}$$

$$F_{\max} = F_2 \quad \text{at} \quad (t_1, t_2) = (t, 0). \tag{45}$$

(v) if $\bar{n}_1^{(\mathrm{in})} \geqslant \bar{n}^{(\mathrm{th})}, \bar{n}_2^{(\mathrm{in})} \geqslant \bar{n}^{(\mathrm{th})}$ and $\Delta\bar{n}_1/\Delta\bar{n}_2 \leqslant \mathrm{e}^{2\kappa t}$,

$$F_{\min} = F_0 \quad \text{at} \quad (t_1, t_2) = (t_0, t - t_0) \tag{46}$$

$$F_{\max} = \begin{cases} F_1 & \text{at} \quad (t_1, t_2) = (0, t) \quad \text{for} \quad \bar{n}_1^{(\mathrm{in})} \geqslant \bar{n}_2^{(\mathrm{in})} \\ F_2 & \text{at} \quad (t_1, t_2) = (t, 0) \quad \text{for} \quad \bar{n}_1^{(\mathrm{in})} < \bar{n}_2^{(\mathrm{in})}. \end{cases} \tag{47}$$

(vi) if $\bar{n}_1^{(\mathrm{in})} < \bar{n}^{(\mathrm{th})}, \bar{n}_2^{(\mathrm{in})} < \bar{n}^{(\mathrm{th})}$ and $\Delta\bar{n}_1/\Delta\bar{n}_2 \leqslant \mathrm{e}^{2\kappa t}$,

$$F_{\min} = \begin{cases} F_2 & \text{at} \quad (t_1, t_2) = (t, 0) \quad \text{for} \quad \bar{n}_1^{(\mathrm{in})} \geqslant \bar{n}_2^{(\mathrm{in})} \\ F_1 & \text{at} \quad (t_1, t_2) = (0, t) \quad \text{for} \quad \bar{n}_1^{(\mathrm{in})} < \bar{n}_2^{(\mathrm{in})} \end{cases} \tag{48}$$

$$F_{\max} = F_0 \quad \text{at} \quad (t_1, t_2) = (t_0, t - t_0). \tag{49}$$

The parameters appearing in (i)–(vi) are given by

$$F_1 = \bar{n}_1^{(\mathrm{in})}(\bar{n}^{(\mathrm{th})}\,\mathrm{e}^{2\kappa t} + \Delta\bar{n}_2) \tag{50}$$

$$F_2 = (\bar{n}^{(\mathrm{th})}\,\mathrm{e}^{2\kappa t} + \Delta\bar{n}_1)\bar{n}_2^{(\mathrm{in})} \tag{51}$$

$$F_0 = \left(\bar{n}^{(\mathrm{th})}\,\mathrm{e}^{\kappa t} + \sqrt{\Delta\bar{n}_1\Delta\bar{n}_2}\right)^2 \tag{52}$$

$$t_0 = \frac{1}{2}t + \frac{1}{4\kappa}\ln\left(\frac{\Delta\bar{n}_1}{\Delta\bar{n}_2}\right). \tag{53}$$

For the distant users to share the inseparable Gaussian state, the inequality $F(t_1, t_2) < |\bar{m}^{(\mathrm{in})}|^2$ must be satisfied. For this purpose, the generator of the initial bipartite Gaussian state should be placed at the site where the function $F(t_1, t_2)$ takes the minimum value $F_{\min}$. In the case (v), the generator of the initial Gaussian state should be placed between the distant users

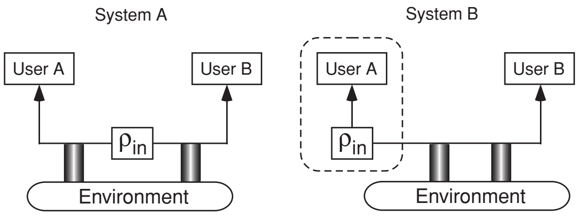

Figure 1. Schematic representation of the symmetric and asymmetric systems (system A and system B) interacting with the noisy environment.

(see system A in figure 1) while in the other cases, it should be placed at the site of the user (see system B in figure 1).

To obtain a better understanding of the results, we suppose that the initial Gaussian state to be shared is symmetric. Since the equality $\bar{n}_1^{(\text{in})} = \bar{n}_2^{(\text{in})} (\equiv \bar{n}^{(\text{in})})$ holds in this case, the parameters F_0, F_1, F_2 and t_0 become

$$F_1 = F_2 = \bar{n}^{(\text{in})}[\bar{n}^{(\text{th})}(\mathrm{e}^{2\kappa t} - 1) + \bar{n}^{(\text{in})}] \tag{54}$$

$$F_0 = [\bar{n}^{(\text{th})}(\mathrm{e}^{\kappa t} - 1) + \bar{n}^{(\text{in})}]^2 \tag{55}$$

$$t_0 = \tfrac{1}{2}t. \tag{56}$$

Then we obtain the minimum and maximum values $F_{\min}$ and $F_{\max}$ of the function $F(t_1, t_2)$:

(i) if $\bar{n}^{(\text{in})} \geqslant \bar{n}^{(\text{th})}$,

$$F_{\min} = F_0 \qquad \text{at} \quad t_1 = t_2 = t/2 \tag{57}$$

$$F_{\max} = F_1 = F_2 \qquad \text{at} \quad (t_1, t_2) = (0, t) \quad \text{or} \quad (t, 0). \tag{58}$$

(ii) If $\bar{n}^{(\text{in})} < \bar{n}^{(\text{th})}$,

$$F_{\min} = F_1 = F_2 \qquad \text{at} \quad (t_1, t_2) = (0, t) \quad \text{or} \quad (t, 0) \tag{59}$$

$$F_{\max} = F_0 \qquad \text{at} \quad t_1 = t_2 = t/2. \tag{60}$$

Hence the necessary and sufficient condition that the distant users can share the inseparable Gaussian state is given by

$$[\bar{n}^{(\text{th})}(\mathrm{e}^{\kappa t} - 1) + \bar{n}^{(\text{in})}]^2 < |\bar{m}^{(\text{in})}|^2 \tag{61}$$

for $\bar{n}^{(\text{in})} \geqslant \bar{n}^{(\text{th})}$ and

$$\bar{n}^{(\text{in})}[\bar{n}^{(\text{th})}(\mathrm{e}^{2\kappa t} - 1) + \bar{n}^{(\text{in})}] < |\bar{m}^{(\text{in})}|^2 \tag{62}$$

for $\bar{n}^{(\text{in})} < \bar{n}^{(\text{th})}$. In particular, since the equality $|\bar{m}^{(\text{in})}|^2 = \bar{n}^{(\text{in})}(\bar{n}^{(\text{in})} + 1)$ holds when the initial Gaussian state is pure, inequality (61) yields

$$2\bar{n}^{(\text{th})}(\mathrm{e}^{\kappa t} - 1) < 1 \qquad \bar{n}^{(\text{in})} > \frac{[\bar{n}^{(\text{th})}(\mathrm{e}^{\kappa t} - 1)]^2}{1 - 2\bar{n}^{(\text{th})}(\mathrm{e}^{\kappa t} - 1)} \qquad (\bar{n}^{(\text{in})} \geqslant \bar{n}^{(\text{th})}) \tag{63}$$

and inequality (62) becomes

$$\bar{n}^{(\text{th})}(\mathrm{e}^{2\kappa t} - 1) < 1 \qquad (\bar{n}^{(\text{in})} < \bar{n}^{(\text{th})}). \tag{64}$$

Therefore when $\bar{n}^{(\text{in})} < \bar{n}^{(\text{th})}$, the distant users can share the inseparable Gaussian state if inequality (64) is satisfied, where the generator of the initial Gaussian state must be placed at the site of the user. On the other hand, when $\bar{n}^{(\mathrm{in})} \geqslant \bar{n}^{(\mathrm{th})}$, the inseparable Gaussian state can be shared if inequality (63) is satisfied, where the generator of the initial Gaussian state must be placed at the mid point between the distant users. The results imply that for bipartite Gaussian states which satisfy $\bar{n}^{(\mathrm{th})} \leqslant \bar{n}^{(\mathrm{in})} < |\bar{m}^{(\mathrm{in})}|$, the decoherence of the entanglement caused by the quantum channels becomes minimum when the quantum channels equally affect the two modes. On the other hand, for bipartite Gaussian state which satisfy $\bar{n}^{(\mathrm{in})} < \bar{n}^{(\mathrm{th})}$ and $\bar{n}^{(\mathrm{in})} < |\bar{m}^{(\mathrm{in})}|$, it becomes minimum when only one of the two modes is affected by the quantum channel and the other remains unchanged. Note that the total transmission time is equal in both cases.

3.2. Symmetric and asymmetric quantum teleportation channels

To investigate the influences of symmetric and asymmetric quantum teleportation channels on the inseparability of the Gaussian bipartite state, we denote the left-hand side of inequality (35) as

$$\mathcal{F}(\tau_1, \tau_2) = \left(\Delta_{\bar{n}_1\bar{m}_1}\tau_1 + \bar{n}_1^{(\mathrm{in})}\right)\left(\Delta_{\bar{n}_2\bar{m}_2}\tau_2 + \bar{n}_2^{(\mathrm{in})}\right). \tag{65}$$

We assume that for the distant users to share the bipartite Gaussian state, the quantum teleportation must be performed totally $\tau = \tau_1 + \tau_2$ times. We obtain the maximum and minimum values $\mathcal{F}_{\max}$ and $\mathcal{F}_{\min}$ of the function $\mathcal{F}(\tau_1, \tau_2)$ under the constraint of $\tau = \tau_1 + \tau_2$. Straightforward calculation yields following results:

(i) if $\left|\bar{n}_1^{(\mathrm{in})}\big/\Delta_{\bar{n}_1\bar{m}_1} - \bar{n}_2^{(\mathrm{in})}\big/\Delta_{\bar{n}_2\bar{m}_2}\right| \leqslant \tau$,

$$\mathcal{F}_{\max} = \mathcal{F}_0 \qquad \text{at} \quad (\tau_1, \tau_2) = (\tau_0, \tau - \tau_0) \tag{66}$$

$$\mathcal{F}_{\min} = \mathcal{F}_1 \qquad \text{at} \quad \begin{cases} (\tau_1, \tau_2) = (0, \tau) & \text{for} \quad \dfrac{\bar{n}_1^{(\mathrm{in})}}{\Delta_{\bar{n}_1\bar{m}_1}} \leqslant \dfrac{\bar{n}_2^{(\mathrm{in})}}{\Delta_{\bar{n}_2\bar{m}_2}} \\ (\tau_1, \tau_2) = (\tau, 0) & \text{for} \quad \dfrac{\bar{n}_1^{(\mathrm{in})}}{\Delta_{\bar{n}_1\bar{m}_1}} > \dfrac{\bar{n}_2^{(\mathrm{in})}}{\Delta_{\bar{n}_2\bar{m}_2}} \end{cases} \tag{67}$$

(ii) If $\left|\bar{n}_1^{(\mathrm{in})}\big/\Delta_{\bar{n}_1\bar{m}_1} - \bar{n}_2^{(\mathrm{in})}\big/\Delta_{\bar{n}_2\bar{m}_2}\right| > \tau$,

$$a\mathcal{F}_{\max} = \mathcal{F}_2 \qquad \text{at} \quad \begin{cases} (\tau_1, \tau_2) = (0, \tau) & \text{for} \quad \dfrac{\bar{n}_1^{(\mathrm{in})}}{\Delta_{\bar{n}_1\bar{m}_1}} - \dfrac{\bar{n}_2^{(\mathrm{in})}}{\Delta_{\bar{n}_2\bar{m}_2}} \geqslant \tau \\ (\tau_1, \tau_2) = (\tau, 0) & \text{at} \quad \dfrac{\bar{n}_2^{(\mathrm{in})}}{\Delta_{\bar{n}_2\bar{m}_2}} - \dfrac{\bar{n}_1^{(\mathrm{in})}}{\Delta_{\bar{n}_1\bar{m}_1}} > \tau \end{cases} \tag{68}$$

$$\mathcal{F}_{\min} = \mathcal{F}_1 \qquad \text{at} \quad \begin{cases} (\tau_1, \tau_2) = (\tau, 0) & \text{for} \quad \dfrac{\bar{n}_1^{(\mathrm{in})}}{\Delta_{\bar{n}_1\bar{m}_1}} - \dfrac{\bar{n}_2^{(\mathrm{in})}}{\Delta_{\bar{n}_2\bar{m}_2}} \geqslant \tau \\ (\tau_1, \tau_2) = (0, \tau) & \text{at} \quad \dfrac{\bar{n}_2^{(\mathrm{in})}}{\Delta_{\bar{n}_2\bar{m}_2}} - \dfrac{\bar{n}_1^{(\mathrm{in})}}{\Delta_{\bar{n}_1\bar{m}_1}} > \tau. \end{cases} \tag{69}$$

The parameters appearing in (i) and (ii) are given by

$$\mathcal{F}_0 = \frac{1}{4}\Delta_{\bar{n}_1\bar{m}_1}\Delta_{\bar{n}_2\bar{m}_2}\left(\tau + \frac{\bar{n}_1^{(\mathrm{in})}}{\Delta_{\bar{n}_1\bar{m}_1}} + \frac{\bar{n}_2^{(\mathrm{in})}}{\Delta_{\bar{n}_2\bar{m}_2}}\right)^2 - \Delta_{\bar{n}_1\bar{m}_1}\Delta_{\bar{n}_2\bar{m}_2}\epsilon^2 \tag{70}$$

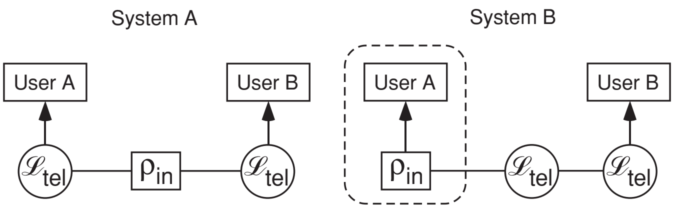

Figure 2. Schematic representation of the symmetric and asymmetric systems (system A and system B) with the quantum teleportation channels. In the figure, $\mathcal{L}_{\rm tel}$ stands for the quantum teleportation channel.

$$\mathcal{F}_1 = \tau \bar{n}_1^{(\rm in)} \bar{n}_2^{(\rm in)} \left(\frac{1}{\tau} + \min \left[\frac{\Delta_{\bar{n}_1 \bar{m}_1}}{\bar{n}_1^{(\rm in)}}, \frac{\Delta_{\bar{n}_2 \bar{m}_2}}{\bar{n}_2^{(\rm in)}} \right] \right) \tag{71}$$

$$\mathcal{F}_2 = \tau \bar{n}_1^{(\rm in)} \bar{n}_2^{(\rm in)} \left(\frac{1}{\tau} + \max \left[\frac{\Delta_{\bar{n}_1 \bar{m}_1}}{\bar{n}_1^{(\rm in)}}, \frac{\Delta_{\bar{n}_2 \bar{m}_2}}{\bar{n}_2^{(\rm in)}} \right] \right) \tag{72}$$

with

$$\epsilon = \tau_0 - \frac{1}{2} \left(\tau + \frac{\bar{n}_2^{(\rm in)}}{\Delta_{\bar{n}_2 \bar{m}_2}} - \frac{\bar{n}_1^{(\rm in)}}{\Delta_{\bar{n}_1 \bar{m}_1}} \right) \tag{73}$$

$$\tau_0 = \mathrm{int} \left[\frac{1}{2} \left(\tau + \frac{\bar{n}_2^{(\rm in)}}{\Delta_{\bar{n}_2 \bar{m}_2}} - \frac{\bar{n}_1^{(\rm in)}}{\Delta_{\bar{n}_1 \bar{m}_1}} \right) \right] \tag{74}$$

where the symbol int$[x]$ stands for the nearest integer to x and thus the parameter ϵ satisfies the inequality $|\epsilon| \leqslant 1/2$. These results together with inequality (35) show that for distant users to share the inseparable Gaussian state, only one mode of the initial Gaussian state should be sent by means of quantum teleportation and the other mode should remain unchanged at the site of the user. For the quantum teleportation channels, it is found that system B in figure 2 is superior to system A for sharing the inseparable Gaussian state.

We suppose that the Gaussian bipartite states used for quantum teleportation are identical and the initial Gaussian state is symmetric. In this case, we have $\Delta_{\bar{n}_1 \bar{m}_1} = \Delta_{\bar{n}_2 \bar{m}_2} \equiv \Delta_{\bar{n}\bar{m}}$ and $\bar{n}_1^{(\rm in)} = \bar{n}_2^{(\rm in)} \equiv \bar{n}^{(\rm in)}$. Furthermore we assume that τ is an even integer for the sake of simplicity. Then the maximum value $\mathcal{F}_{\max}$ of the function $\mathcal{F}(\tau_1, \tau_2)$ is attained when the generator of the initial Gaussian state is placed at the mid point between the distant users, where the maximum value $\mathcal{F}_{\max}$ is given by $\mathcal{F}_{\max} = (\bar{n}^{(\rm in)} + \tau \Delta_{\bar{n}\bar{m}}/2)^2$. On the other hand, the minimum value $\mathcal{F}_{\min}$ is attained when the generator is placed at the site of the user, where $\mathcal{F}_{\min}$ is given by $\mathcal{F}_{\min} = \bar{n}^{(\rm in)}(\bar{n}^{\rm in} + \tau \Delta_{\bar{n}\bar{m}})$. Therefore the condition for the distant users to share the inseparable Gaussian state is given by $\bar{n}^{(\rm in)}(\bar{n}^{(\rm in)} + \tau \Delta_{\bar{n}\bar{m}}) < |\bar{m}^{(\rm in)}|^2$, where the generator of the initial Gaussian state must be held by one of the distant users. In particular, when the initial Gaussian state is pure, where the inequality $|\bar{m}^{(\rm in)}|^2 = \bar{n}^{(\rm in)}(\bar{n}^{(\rm in)} + 1)$ holds, we obtain the condition $\tau \Delta_{\bar{n}\bar{m}} < 1$ which does not depend on the parameter $\bar{n}^{(\rm in)}$ of the initial Gaussian state. When the bipartite Gaussian states used for the quantum teleportation are separable, where the inequality $\bar{n} \geqslant |\bar{m}|$ holds, we obtain the inequality $\Delta_{\bar{n}\bar{m}} \geqslant 1$ and thus the inseparability condition is not satisfied. This gives the trivial fact that the entanglement is indispensable for teleporting the quantum nature.

4. Quantum communication in noisy quantum channels

4.1. Continuous variable quantum teleportation

This section investigates the properties of continuous variable quantum teleportation with the standard protocol [5, 15–18] when the sender and receiver share a two-mode squeezed-vacuum state by means of the symmetric or asymmetric quantum channels. When the sender teleports an unknown quantum $\hat{\rho}_{\rm in}$, the receiver obtains the quantum state $\hat{\rho}_{\rm out}$ which is related to the input state $\hat{\rho}_{\rm in}$ by [17, 18]

$$\hat{\rho}_{\rm out} = \int \frac{{\rm d}^2\alpha}{\pi} P(\alpha)\hat{D}(\alpha)\hat{\rho}_{\rm in}\hat{D}^\dagger(\alpha) \tag{75}$$

with

$$P(\alpha) = \int \frac{{\rm d}^2\beta}{\pi} W(\beta^* - \alpha^*, \beta) \tag{76}$$

where $W(\alpha, \beta)$ is the Wigner function of the bipartite quantum state shared by the sender and receiver. The Wigner function $W_r(z_1, z_2)$ of a two-mode squeezed-vacuum state $|\Psi_r\rangle = {\rm e}^{r(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2)}|0, 0\rangle$ with the squeezing parameter $r > 0$ is given by

$$W_r(z_1, z_2) = 4\exp[-2(|z_1|^2 + |z_2|^2)\cosh 2r + 2(z_1z_2 + z_1^*z_2^*)\sinh 2r]. \tag{77}$$

We suppose that the mode 1 (2) of the two-mode squeezed-vacuum state is sent to the sender (the receiver) through the linear dissipative channel, the transmission time of which is τ_1 (τ_2). Then using the function $G_k(z|z'; t)$ given by equation (4), we can calculate the Wigner function $W(z_1, z_2)$ of the bipartite quantum state shared by the sender and receiver,

$$\begin{aligned} W(z_1, z_2) &= \int \frac{{\rm d}^2z_1'}{\pi}\int\frac{{\rm d}^2z_2'}{\pi} G_1(z_1|z_1'; \tau_1)G_2(z_2|z_2'; \tau_2)W_r(z_1', z_2') \\ &= \frac{4}{\sigma_S}\exp\left[-\frac{\sigma_2}{\sigma_S}|z_1|^2 - \frac{\sigma_1}{\sigma_S}|z_2|^2 + \frac{\lambda}{\sigma_S}(z_1z_2 + z_1^*z_2^*)\right] \end{aligned} \tag{78}$$

where the parameters $\sigma_S, \sigma_1, \sigma_2$ and λ are given by

$$\sigma_S = \tfrac{1}{2}\left(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+\right) \tag{79}$$

$$\sigma_k = \sigma_k^+ + \sigma_k^- \tag{80}$$

$$\lambda = \sqrt{\left(\sigma_1^+ - \sigma_1^-\right)\left(\sigma_2^+ - \sigma_2^-\right)} \tag{81}$$

with

$$\sigma_k^\pm = \left(2\bar{n}_k^{(\rm th)} + 1\right)(1 - {\rm e}^{-2\kappa_k\tau_k}) + {\rm e}^{\pm 2r}\,{\rm e}^{-2\kappa_k\tau_k}. \tag{82}$$

Substituting equation (78) into equation (76), we obtain

$$P(\alpha) = \frac{1}{\sigma}\exp\left(-\frac{|\alpha|^2}{\sigma}\right) \tag{83}$$

with

$$\begin{aligned} \sigma = \left(\bar{n}_1^{(\rm th)} + \frac{1}{2}\right)(1 - {\rm e}^{-2\kappa_1\tau_1}) + \left(\bar{n}_2^{(\rm th)} + \frac{1}{2}\right)(1 - {\rm e}^{-2\kappa_1\tau_1}) \\ + {\rm e}^{-2r}\left(\frac{{\rm e}^{-\kappa_1\tau_1} + {\rm e}^{-\kappa_2\tau_2}}{2}\right)^2 + {\rm e}^{2r}\left(\frac{{\rm e}^{-\kappa_1\tau_1} - {\rm e}^{-\kappa_2\tau_2}}{2}\right)^2. \end{aligned} \tag{84}$$

Then the input–output relation of the quantum teleportation is given by

$$\hat{\rho}_{\text{out}} = \frac{1}{\sigma}\int \frac{d^2\alpha}{\pi}\, e^{-|\alpha|^2/\sigma}\hat{D}(\alpha)\hat{\rho}_{\text{in}}\hat{D}^{\dagger}(\alpha). \tag{85}$$

In this case, the s-parametrized phase-space quasi-distribution functions $F_{\text{in}}^{(s)}(\alpha)$ and $F_{\text{out}}^{(s)}(\alpha)$ [13, 14] which represent the quantum states $\hat{\rho}_{\text{in}}$ and $\hat{\rho}_{\text{out}}$ are related to each other by $F_{\text{out}}^{(s)}(\alpha) = F_{\text{in}}^{(s+2\sigma)}(\alpha)$. When a coherent state $|\alpha\rangle$ is teleported, the fidelity $\mathcal{F}_c = \langle\alpha|\hat{\rho}_{\text{in}}|\alpha\rangle$ is calculated to be

$$\mathcal{F}_c = \frac{1}{1+\sigma}. \tag{86}$$

It is important to note that the parameter σ given by equation (84) does not decrease monotonically with respect to the squeezing parameter r if the quantum channels are asymmetric. In fact, the parameter σ takes the minimum value $\sigma_{\min}$ at $r = r_0$, where r_0 and $\sigma_{\min}$ are given respectively by

$$r_0 = \frac{1}{2}\ln\left|\frac{e^{-\kappa_1\tau_1} + e^{-\kappa_2\tau_2}}{e^{-\kappa_1\tau_1} - e^{-\kappa_2\tau_2}}\right| \tag{87}$$

and

$$\sigma_{\min} = \left(\bar{n}_1^{(\text{th})} + \frac{1}{2}\right)(1 - e^{-2\kappa_1\tau_1}) + \left(\bar{n}_2^{(\text{th})} + \frac{1}{2}\right)(1 - e^{-2\kappa_1\tau_1}) + \frac{1}{2}|e^{-2\kappa_1\tau_1} - e^{-2\kappa_2\tau_2}|. \tag{88}$$

For the symmetric channels, the minimum value $\sigma_{\min} = (2\bar{n}^{(\text{th})} + 1)(1 - e^{-2\kappa t})$ is attained at the limit $r \to \infty$. In particular, when the noisy environments are in their vacuum states, that is, $\bar{n}_1^{(\text{th})} = \bar{n}_2^{(\text{th})} = 0$, we obtain

$$\sigma_{\min} = 1 - \min[e^{-2\kappa_1\tau_1}, e^{-2\kappa_2\tau_2}]. \tag{89}$$

It is found from the result that when the linear dissipative channels are asymmetric, the performance of the continuous variable quantum teleportation with the standard protocol is deteriorated by the strong squeezing ($r > r_0$) while it is improved for the symmetric quantum channels.

When the sender and receiver share the two-mode squeezed-vacuum via the quantum teleportation channels, the Wigner function $W(z_1, z_2)$ of the bipartite quantum state shared by them is calculated to be

$$\begin{aligned} W(z_1, z_2) &= \int \frac{d^2 z_1'}{\pi}\int \frac{d^2 z_2'}{\pi}\mathcal{G}_1(z_1|z_1'; \tau_1)\mathcal{G}_2(z_2|z_2'; \tau_2)W_r(z_1', z_2') \\ &= \frac{4}{\delta_S}\exp\left[-\frac{\delta_2}{\delta_S}|z_1|^2 - \frac{\delta_1}{\delta_S}|z_2|^2 + \frac{\epsilon}{\delta_S}(z_1 z_2 + z_1^* z_2^*)\right] \end{aligned} \tag{90}$$

where the function $\mathcal{G}_k(z|z; \tau_k)$ is given by equation (21) and the parameters δ_S, δ_1 and δ_2 are defined by

$$\delta_S = 1 + 2(\tau_1\Delta_{\bar{n}_1\bar{m}_1} + \tau_2\Delta_{\bar{n}_2\bar{m}_2})\cosh 2r + 4\tau_1\tau_2\Delta_{\bar{n}_1\bar{m}_1}\Delta_{\bar{n}_2\bar{m}_2} \tag{91}$$

$$\delta_k = 2\cosh 2r + 4\tau_k\Delta_{\bar{n}_k\bar{m}_k} \tag{92}$$

$$\epsilon = 2\sinh 2r. \tag{93}$$

In this case, the function $P(\alpha)$ given by equation (76) becomes

$$P(\alpha) = \frac{1}{\delta}\exp\left(-\frac{|\alpha|^2}{\delta}\right) \tag{94}$$

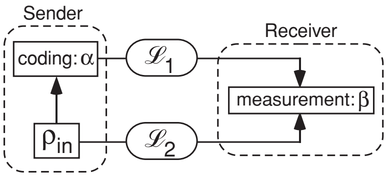

Figure 3. Schematic representation of the continuous variable quantum dense coding system, where $\hat{\mathcal{L}}_k$ ($k = 1, 2$) stands for the quantum channels under the influence of the noisy environment and induced by the quantum teleportation, and α and β represent the encoded symbol and the measurement outcome.

with

$$\delta = \mathrm{e}^{-2r} + \tau_1 \Delta_{\bar{n}_1 \bar{m}_1} + \tau_2 \Delta_{\bar{n}_2 \bar{m}_2}. \tag{95}$$

The fidelity $\mathcal{F}_c = \langle\alpha|\hat{\rho}_{\mathrm{in}}|\alpha\rangle$ of the coherent state $|\alpha\rangle$ is given by $\mathcal{F}_c = 1/(1+\delta)$. It is found from equation (95) that in contrast to the case for the linear dissipative channels, the performance of quantum teleportation improves as the squeezing is made stronger, regardless to whether the quantum channels are symmetric or not.

4.2. Continuous variable quantum dense coding

We next consider the continuous variable quantum dense coding by means of a two-mode squeezed-vacuum state [26, 27] under the influences of the symmetric and asymmetric quantum channels. We assume that the generator of a two-mode squeezed-vacuum state is placed at the sender site. The sender encodes the information by applying the displacement operator $\hat{D}(\alpha)$ on one of the two modes, and the two modes are sent to the receiver through the linear dissipative channels or quantum teleportation quantum channels. After receiving the encoded state, the receiver performs a simultaneous measurement of position and momentum on the two modes, where the measurement result can be represented by the complex parameter $\beta = (x + \mathrm{i}p)/\sqrt{2}$. The quantum dense coding system considered here is depicted in figure 3.

When the sender encodes the information α and the transmission time of the two modes passing through the linear dissipative channels is t, the conditional probability $P(\beta|\alpha)$ that the receiver obtains the measurement outcome β is given by

$$P(\beta|\alpha) = \frac{1}{\pi\sigma_t} \exp\left(-\frac{|\beta - \alpha\,\mathrm{e}^{-\kappa_1 t}|^2}{\sigma_t}\right) \tag{96}$$

with

$$\sigma_t = \left(\bar{n}_1^{(\mathrm{th})} + \frac{1}{2}\right)(1 - \mathrm{e}^{-2\kappa_1 t}) + \left(\bar{n}_2^{(\mathrm{th})} + \frac{1}{2}\right)(1 - \mathrm{e}^{-2\kappa_1 t}) + \mathrm{e}^{-2r}\left(\frac{\mathrm{e}^{-\kappa_1 t} + \mathrm{e}^{-\kappa_2 t}}{2}\right)^2 + \mathrm{e}^{2r}\left(\frac{\mathrm{e}^{-\kappa_1 t} - \mathrm{e}^{-\kappa_2 t}}{2}\right)^2. \tag{97}$$

When the prior probability $P_{\mathrm{in}}(\alpha)$ that the sender encodes α is the Gaussian distribution

$$P_{\mathrm{in}}(\alpha) = \frac{1}{\pi\bar{n}^{(\mathrm{in})}} \exp\left(-\frac{|\alpha|^2}{\bar{n}^{(\mathrm{in})}}\right) \tag{98}$$

the output probability $P_{\mathrm{out}}(\beta)$ that the receiver obtains β as the measurement outcome is given by

$$P_{\mathrm{out}}(\beta) = \int \mathrm{d}^2\alpha\, P(\beta|\alpha) P_{\mathrm{in}}(\alpha) = \frac{1}{\pi[\sigma_t + \bar{n}^{(\mathrm{in})}\,\mathrm{e}^{-2\kappa_1 t}]} \exp\left(-\frac{|\beta|^2}{\sigma_t + \bar{n}^{(\mathrm{in})}\,\mathrm{e}^{-2\kappa_1 t}}\right). \tag{99}$$

Then the mutual information of the quantum dense coding system is calculated to be

$$I = \int d^2\alpha \int d^2\beta\, P_{\rm in}(\alpha) P(\beta|\alpha) \log\left[\frac{P(\beta|\alpha)}{P_{\rm out}(\beta)}\right] = \log\left(1 + \frac{\bar{n}^{(\rm in)}\, e^{-2\kappa_1 t}}{\sigma_t}\right). \tag{100}$$

It is found from equations (97) and (100) that when the squeezing parameter r takes the value $r_0 = (1/2)\ln|(e^{-\kappa_1 t} + e^{-\kappa_2 t})/(e^{-\kappa_1 t} - e^{-\kappa_2 t})|$, the rate of the information transmission in the quantum dense coding system becomes maximum.

On the other hand, when each mode of the encoded two-mode squeezed-vacuum state is transmitted through the quantum teleportation channel, the condition probability $P(\beta|\alpha)$ is given by

$$P(\beta|\alpha) = \frac{1}{\pi\delta_\tau} \exp\left(-\frac{|\beta-\alpha|^2}{\delta_\tau}\right) \tag{101}$$

with

$$\delta_\tau = e^{-2r} + \tau(\Delta_{\bar{n}_1\bar{m}_1} + \Delta_{\bar{n}_2\bar{m}_2}). \tag{102}$$

In this case, the mutual information of the quantum dense coding system becomes

$$I = \log\left(1 + \frac{\bar{n}^{\rm in)}}{\delta_\tau}\right) \tag{103}$$

which means that the information transmission rate increases monotonically as the value of the squeezing parameter r is increased.

5. Concluding remarks

We have investigated the influences of the linear dissipative channel and the quantum teleportation channels on quantum entanglement. In particular, we have focused our attention on the asymmetry of the quantum channels. When a bipartite Gaussian state is shared through the linear dissipative channels, the asymmetric or symmetric channel is suitable for sharing the entanglement, depending on the channel parameters and the initial Gaussian state. On the other hand, when the quantum teleportation channel is applied for sharing the bipartite quantum state, the asymmetric channel is always better than the symmetric channel. Furthermore, we have investigated quantum teleportation and quantum dense coding of continuous variables, where the two-mode squeezed-vacuum state is used as the entanglement resource. When the two-mode squeezed-vacuum state is shared by means of the linear dissipative channels, increasing the squeezing makes the fidelity higher in quantum teleportation and the information rate greater in quantum dense coding for the symmetric channels while there is the optimum value of the squeezing for the asymmetric quantum channels. If the squeezing parameter takes values greater than the optimum one, quantum teleportation and quantum dense coding are deteriorated. On the other hand, when the two-mode squeezed-vacuum state is shared by means of the quantum teleportation channels, increasing the squeezing always makes the fidelity higher and the information rate greater. In this paper, we have considered the linear dissipative channel and the quantum teleportation channel for bosonic systems. It is important to investigate the influences of asymmetric quantum channels on the quantum entanglement of finite dimensional systems.

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